UCLA-00-TEP-08
LAPTH 784-00
February 2000



# Muon colliders and the non-perturbative dynamics of the Higgs boson[1]

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A muon collider operating in the TeV energy range can be an ideal $s$-channel Higgs boson factory. This is especially true for a very heavy Higgs boson. The non-perturbative dynamical aspects of such a Higgs boson were recently investigated with large $N$ expansion methods at next to leading order, and reveal the existence of a mass saturation effect. Even at strong coupling, the Higgs resonance remains always below 1 TeV. However, if the coupling is strong enough, the resonance becomes impossible to be detected.

[1] Invited talk presented by A. Ghinculov at the $5^{th}$ International Conference on Physics Potential and Development of $\mu^+\mu^-$ Colliders, December 15-17, 1999, Fairmont Hotel, San Francisco, CA, USA.

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A muon collider operating in the TeV energy range can be an ideal $s$-channel Higgs boson factory. This is especially true for a very heavy Higgs boson. The non-perturbative dynamical aspects of such a Higgs boson were recently investigated with large $N$ expansion methods at next to leading order, and reveal the existence of a mass saturation effect. Even at strong coupling, the Higgs resonance remains always below 1 TeV. However, if the coupling is strong enough, the resonance becomes impossible to be detected.

A central question in today's particle physics is how the electroweak symmetry breaking is realized in nature. Further experimental input is needed for distinguishing between various theoretical possibilities, and this will be the main goal of the LHC. The simplest of these possibilities is the minimal scalar sector of the standard model which predicts the existence of one single Higgs particle.

The sensitivity of low energy quantum corrections to the mass of the Higgs boson is small because of Veltman's screening theorem. Therefore the indirect Higgs mass determination from radiative corrections is rather imprecise, in spite of the impressive accuracy of LEP, SLC, and Tevatron measurements. Current electroweak data fits based on the minimal standard model favor a lighter Higgs boson, with a central value around 110 GeV, which is close to the region excluded by direct production bounds.

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So far no significant deviations from the standard model radiative corrections were measured which would hint towards the existence of additional degrees of freedom at higher energy. However, their existence is strongly supported by well-known open questions of the standard model on the theoretical side. Such degrees of freedom have the potential to induce additional radiative corrections and thus shift the prediction for the Higgs boson mass. It is conceivable that, once built, the LHC will discover a Higgs resonance considerably heavier than the central values suggested by electroweak data fits at present.

An interesting feature of a possible muon collider is that it can be used as an $s$-channel Higgs factory. Here we would like to discuss the implications of the non-perturbative dynamics of the scalar sector for $\mu^+\mu^-$ Higgs factories. We will argue that due to the non-perturbative dynamics of the scalar sector, a possible muon collider will not need an energy much higher than 1 TeV to study even a strongly coupled standard Higgs boson. However, it may need a high luminosity.

A heavy Higgs boson implies a strongly self-interacting scalar sector. Thus it complicates the theoretical analysis because at some point perturbation theory becomes unreliable. A few radiative corrections induced by heavy Higgs bosons are available in higher order [1]. Their convergence properties were studied by several authors [2], and revealed rather large theoretical uncertainties. In order to avoid the problems of perturbation theory at strong coupling, such as large renormalization scheme uncertainties and the blow-up of radiative corrections in higher loop order, a non-perturbative approach is necessary.

We performed a study of the Higgs sector at strong coupling by using non-perturbative $1/N$ expansion techniques at higher order. This study revealed the existence of an interesting mass saturation effect. When the coupling constant of the scalar sector is increased, the mass of the Higgs boson remains bounded under a saturation value just under 1 TeV, while its widths continues to increase.

Along the lines of 't Hooft's work on planar QCD [3], the large $N$ expansion has attracted a lot of attention by holding the promise to solve nonabelian gauge theories non-perturbatively. It was also used in the study of critical phenomena. Its connections to matrix models, two-dimensional gravity, and string theory were also explored.

Given that the standard model's Higgs sector is a gauged $SU(2)$ sigma model, the $1/N$ expansion suggests itself naturally for studying it at strong coupling. At leading order in $1/N$, this was initiated in ref. [4]. Unfortunately, the leading order solution proves to be quite a poor approximation, which in the weak coupling limit deviates substantially from perturbation theory. Because of this it cannot be used in realistic phenomenological studies. In ref. [5] we extended this study to next-to-leading order. It turns out that the next-to-leading order solution is impressively accurate. In the weak coupling limit it competes with the best perturbative results available at two-loop precision.

The starting point of the $1/N$ analysis is the Lagrangean of the standard model's scalar sector promoted to a $O(N)$-symmetric sigma model. The well-known equiv-

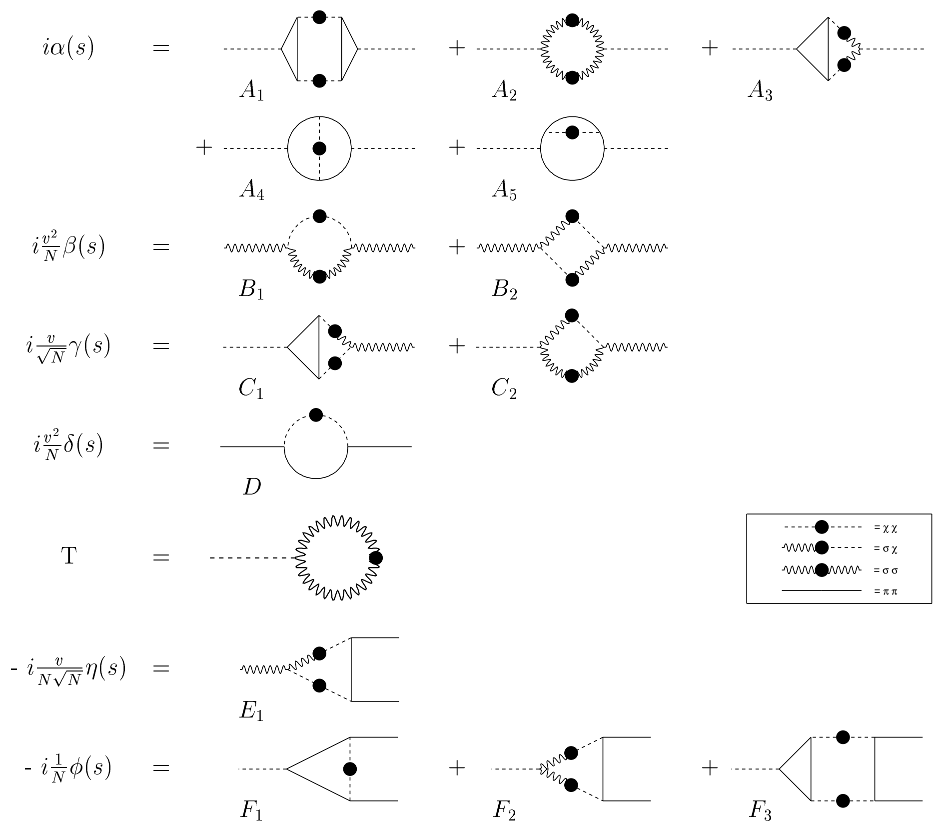


Figure 1: *Multiloop diagrams which contribute in next-to-leading order in* $1/N$ *to the two- and three-point functions of the* $O(N)$ *sigma model. The blobs on propagators denote chains of one-loop Goldstone boson bubble diagrams.*

alence theorem provides a relation between the physics of the purely scalar sector and the physics of electroweak vector bosons. The standard model case is recovered in the $N = 4$ limit:

$$\mathcal{L}_1 = \frac{1}{2}\partial_\nu \Phi_0 \partial^\nu \Phi_0 - \frac{\mu_0^2}{2}\Phi_0^2 - \frac{\lambda_0}{4!N}\Phi_0^4 \quad , \quad \Phi_0 \equiv \left(\phi_0^1, \phi_0^2, \ldots, \phi_0^N\right) \tag{1}$$

The next step is to introduce an additional unphysical field $\chi$ in this Lagrangian [4]:

$$\begin{aligned} \mathcal{L}_2 &= \mathcal{L}_1 + \frac{3N}{2\lambda_0}(\chi_0 - \frac{\lambda_0}{6N}\Phi_0^2 - \mu_0^2)^2 \\ &= \frac{1}{2}\partial_\nu \Phi_0 \partial^\nu \Phi_0 - \frac{1}{2}\chi_0 \Phi_0^2 + \frac{3N}{2\lambda_0}\chi_0^2 - \frac{3\mu_0^2 N}{\lambda_0}\chi_0 + const. \end{aligned} \tag{2}$$

The auxiliary field $\chi$ does not correspond to a dynamical degree of freedom. Its equation of motion is simply a constraint and can be used for eliminating $\chi$.

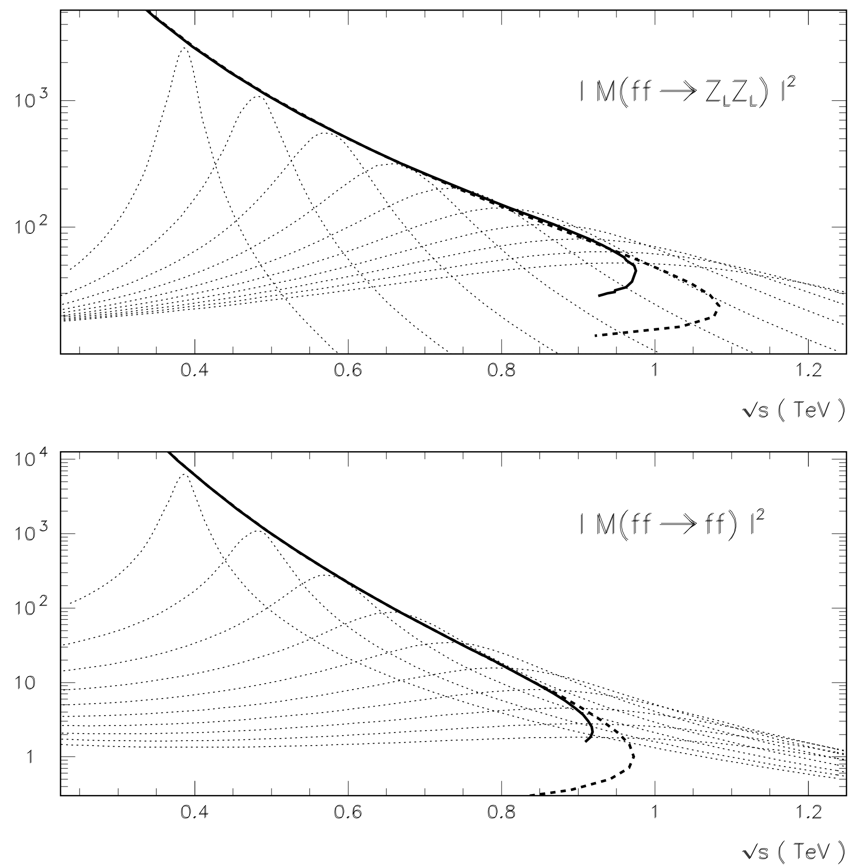


Figure 2: *The Higgs line shape at a $\mu^+\mu^-$ Higgs factory. We marked the position of the maxima of the resonances (solid line for the $1/N$ result and dashed line for the perturbative result at two-loop).*

While the introduction of the auxiliary field does not change the dynamics, it does alter the Feynman rules by eliminating the scalar quartic couplings. This proves to be extremely helpful for calculations beyond leading order in $1/N$. Denoting the Higgs boson by $\sigma$ and the Goldstone bosons by $\pi$, the Feynman rules derived from the Lagrangean $\mathcal{L}_2$ have only trilinear vertices of the type $\chi\sigma\sigma$ and $\chi\pi\pi$. One can easily count the powers of $N$ of a Feynman graph by noticing that closed Goldstone loops give rise to a factor $N$, $\chi\chi$ propagators have a factor $1/N$, and mixed $\chi\sigma$ propagators have a factor $1/\sqrt{N}$.

In figure 1 we show the Feynman diagrams which we need for calculating Higgs production and decay processes at muon colliders at next-to-leading order in the $1/N$ expansion. These are all one-, two-, and three-point functions of the sigma model. We note that the summation of leading order renormalon chains on internal

propagators of these diagrams leads to an additional Euclidean pole in the propagators. The origin and physical content of the tachyon pole is discussed in ref. [5]. When effectively calculating the diagrams in figure 1, we use the minimal tachyonic subtraction discussed in ref. [5] for treating it.

We calculated the diagrams shown in figure 1 numerically, along the lines of ref. [6]. Once they are available numerically, they can be used for deriving amplitudes of physical processes. Two Higgs processes are of interest at $\mu^+\mu^-$ $s$-channel Higgs factories: $\mu^+\mu^- \to H \to t\bar{t}$ and $\mu^+\mu^- \to H \to ZZ, W^+W^-$. Their amplitudes are given in the $1/N$ expansion by the following expression at next-to-leading order:

$$\begin{aligned}
\mathcal{M}_{f\bar{f}} &= \frac{1}{s - m^2(s)\left[1 - \frac{1}{N} f_1(s)\right]} \\
\mathcal{M}_{WW} &= \frac{m^2(s)}{\sqrt{N} v} \frac{1 - \frac{1}{N} f_2}{s - m^2(s)\left[1 - \frac{1}{N} f_1(s)\right]}
\end{aligned} \tag{3}$$

Here, the correction functions $f_1$ and $f_2$ are given by a combination of the two- and three-point functions defined in figure 1:

$$\begin{aligned}
f_1(s) &= \frac{m^2(s)}{v^2}\,\hat{\alpha}(s) + 2\hat{\gamma}(s) + \frac{v^2}{m^2(s)}\left[\hat{\beta}(s) - 2\frac{s - m^2(s)}{v^2}\left(\delta Z_\sigma - \delta Z_\pi\right)\right] \\
f_2(s) &= \frac{m^2(s)}{v^2}\,\hat{\alpha}(s) + \hat{\gamma}(s) - \hat{\phi}(s) - \frac{v^2}{m^2(s)}\hat{\eta}(s)
\end{aligned} \tag{4}$$

The wave function renormalizations $\delta Z_\sigma$, $\delta Z_\pi$ can be extracted from $\hat{\beta}$, $\hat{\gamma}$. The hat in the expressions above means that the multi-loop diagrams are subtracted recursively in the ultraviolet, according to the Bogoliubov-Parasiuk-Hepp-Zimmermann procedure [5]. We note that by performing these ultraviolet subtractions we introduce a renormalization scale. However, in the final physical correction functions $f_1$ and $f_2$ this renormalization scheme dependence cancels out. The final result in manifestly independent of the choice of the renormalization scheme.

In figure 2 we show numerical results for the $\mu^+\mu^- \to H \to t\bar{t}$ and $\mu^+\mu^- \to H \to ZZ, W^+W^-$ processes of eqs 3. In both processes the Higgs mass saturation effect shows up. When the strength of the coupling increases, the peak of the resonance shifts towards higher energy, up to a saturation value just under 1 TeV, and then starts to shift back towards lower energy. At the same time, the width continues to increase and the resonance becomes flat and difficult do detect experimentally.

To conclude, we performed a non-perturbative study of the two main Higgs processes of interest at a future muon collider. Due to the non-perturbative dynamics of the Higgs sector, a standard Higgs particle is bound to result into a resonance with a peak below 1 TeV. Therefore, a muon collider will not need energies much larger than 1 TeV to cover the whole range where a standard Higgs may exist. However,

due to non-perturbative dynamics, at strong coupling the experimental detection becomes difficult. To measure a flat Higgs resonance will require precise knowledge of the backgrounds. Detection will be a matter of luminosity and not of center of mass energy.

Finally, if the coupling becomes strong enough, the Higgs boson will still remain under 1 TeV but will become impossible to detect with a given luminosity.

**Aknowledgement** The work of A.G. was supported by the US Department of Energy. The work of T.B. was supported by the EU Fourth Training Programme "Training and Mobility of Researchers", Network "Quantum Chromodynamics and the Deep Structure of Elementary Particles", contract FMRX-CT98-0194 (DG 12 - MIHT).

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